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Large Modal Survey Testing Using the Ibrahim Time Domain Identification Technique

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The ability of the Ibrahim time domain identification algorithm to identify a complete set of structural modal parameters, using a large number of free-response time histories simultaneously in one analysis and assuming an identification model with a high number of degrees of freedom, has been studied. Identification results using simulated free responses of a uniform rectangular plate, with 225 measurement stations, and experimental responses from a ground vibration test of the long duration exposure facility (LDEF) Space Shuttle payload, with 142 measurement stations, are presented. As many as 300 degrees of freedom were allowed in analyzing these data. In general, the use of a significantly oversized identification model in the identification process was found to maintain or increase identification accuracy and to identify modes of low response level that are not identified with smaller identification model sizes. The concept of a mode shape correlation constant is introduced for use when more than one identification analysis of the same structure are conducted. This constant quantifies the degree of correlation between any two sets of complex mode shapes identified, using different excitation conditions, different user-selectable algorithm constants, or overlapping sets of measurements.

Nomenclature

$[A]$	= square system matrix (of order $2m$)
ITD	= Ibrahim time domain (technique)
m	= number of computational degrees of freedom (NDOF)
MCF	= modal confidence factor
MSCC	= mode shape correlation constant
NDOF	= number of computational degrees of freedom
N/S	= noise-to-signal ratio
$\{n(t)\}$	= vector of measurement noise time histories
OAMCF	= overall modal confidence factor
p	= number of structural modes
rms	= root mean square
s	= number of rows in $[\Phi]$ and $[\hat{\Phi}]$
t_j	= time instant j
$\{x(t)\}$	= vector of free-response time histories
α_k	= kth eigenvalue of $[A]$
$\{\gamma\}_k$	= portion of $\{\psi\}_k$
$(\Delta t)_l$	= time shift between $[\Phi]$ and $[\hat{\Phi}]$
ζ	= modal damping factor = C/C_c
λ_k	= kth characteristic root of structure
$[\Phi]$, $[\hat{\Phi}]$	= response matrices ($2m \times s$)
$\{\psi\}_k$	= kth eigenvector (complex mode shape) of $[A]$
$\{\ \}^T$	= transpose of vector $\{\ \}$
$\{\ \}^*$	= complex conjugate of vector $\{\ \}$
$\|\ \|$	= magnitude

Introduction

USING a time-domain approach, it has been shown that the identification of structural modal parameters from experimental data can be placed in the form of a complex eigenvalue problem.[1] The resulting method, referred to as the Ibrahim time domain (ITD) technique, uses free-response time histories $\{x(t)\}$ measured at various points on a test structure to compute a square system matrix $[A]$, or order $2m$, in a least-squares sense from the equation

$$[A][\Phi\Phi^T] = [\hat{\Phi}\Phi^T] \tag{1}$$

In this equation, $[\Phi]$ and $[\hat{\Phi}]$ are rectangular matrices of size $2m \times s$, with $s \geq 2m$, whose elements are

$$\Phi_{ij} = x_i(t_j) \qquad \hat{\Phi}_{ij} = x_i[t_j + (\Delta t)_l] \tag{2}$$

The ith row of $[\Phi]$ corresponds to the ith measurement or a measurement delayed some arbitrary time, $\Delta\tau$. The use of delayed or "transformed" stations[1] allows the computation of a modal confidence factor[2] (MCF) at each station for each identified mode. The MCF parameter is used to differentiate the desired structural modes from "noise modes," computed whenever the number of structural modes contributing to the responses is smaller than m. A (complex valued) MCF is calculated for each station, and indicates the consistency of the modal deflection identified at each station with the deflection at the same station identified using data measured a small time later. Its value is near 100% in amplitude and 0 deg in phase for accurately identified structural modes.

Possible time-domain functions which can be used include actual free decays obtained following random excitation of the structure, unit-impulse response functions calculated by the inverse Fourier transform of frequency response functions, or "random-decrement" functions[3] calculated from random operating time histories.

After computing $[A]$ from Eq. (1), an eigenvalue problem of the form

$$[A]\{\psi\}_k = \alpha_k\{\psi\}_k \tag{3}$$

is solved. The kth eigenvector of $[A]$ is the kth complex mode shape of the structure and the kth eigenvalue of $[A]$ is related to the structure's characteristic root λ_k through the equation

$$\alpha_k = e^{\lambda_k (\Delta t)_l} \tag{4}$$

Details of the identification technique are contained in Refs. 1-4.

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In previous applications of the technique using simulated and experimental data,[4-6] the primary purpose was to establish credibility of the method. The studies were limited, for research purposes, to problems with small numbers of measurement stations and structural modes. For large modal survey tests, however, it is not unusual to obtain responses at 200 or more measurement stations on a test structure.

If the response functions used in the identification technique are entirely noise-free, the size of the identification model, m (or "number of allowed computational degrees of freedom") must exactly equal the number of excited structural modes in the responses. When experimental data are processed, however, there is always some level of noise, and the number of structural modes contributing to the responses is not exactly known. Since the maximum number of modes identified in a single computer analysis is equal to the number of allowed degrees of freedom, an obvious question is whether all parameters can be accurately obtained by simply allowing the number of degrees of freedom to be unquestionably larger than the number of excited structural modes. Since the number of measurements analyzed in each identification run can be as high as the number of allowed degrees of freedom, the use of a large enough identification model would also allow the response functions for all stations selected for the modal survey to be processed simultaneously.

When the number of available measurements is less than the number of degrees of freedom desired in the identification process, extra "transformed stations" can be formed by delaying the original response functions by small arbitrary time increments. Using this approach, a large identification model can be used even when a relatively small number of responses are available.

If computer storage limitations restrict the processing of a large number of response measurements simultaneously, a technique is presented for correlating sets of identified mode shapes from different runs using data corresponding to a common set of measurements used in each analysis. With this approach, modes obtained for two or more portions of the available test measurements can be matched more accurately than on the basis of identified frequencies and damping values alone. A mode shape correlation constant (MSCC), whose value is zero for no correlation and 1.0 for complete correlation, is introduced for this purpose. The MSCC is a general procedure for measuring the degree of correlation between any two complex modal vectors. It has also been found useful in correlating mode shapes identified with responses from different excitation conditions or identified using different values of the few user-selectable algorithm constants, to provide additional confidence in the identification results by studying the consistency of independent analyses.

This paper presents typical results that have been obtained in processing simulated and experimental data with the ITD identification algorithm using a large number of measurement stations and/or large identification model sizes. The results indicate that the use of large identification models in the analysis procedure can result in the accurate identification of a large number of structural modes in a single analysis, often allowing all measured response data from a large model survey test to be used simultaneously in computing the modal parameters of a test structure.

Theory of Oversized Identification Model

A set of free-response functions containing modal information from p structural modes of vibration can be expressed as

$$\{x(t)\} = \sum_{k=1}^{2p} \{\psi\}_k e^{\lambda_k t} \tag{5}$$

If noise-free responses are used in the identification algorithm, the identification model must have exactly p degrees of freedom for unique identification. If more than p degrees of freedom are allowed, the $\{\Phi\}$ matrix is singular.

In experimental work, however, measured responses always contain a certain amount of noise. These responses can be expressed as

$$\{x(t)\} = \sum_{k=1}^{2p} \{\psi\}_k e^{\lambda_k t} + \{n(t)\} \tag{6}$$

In previous applications[1-6] it was found that using noisy responses in the identification process with the number of degrees of freedom larger than p yielded good results without encountering singularity. The results even improved as the identification model size was increased. The qualitative explanation for this situation is that the extra degrees of freedom act as outlets for the noise. In this case, the noisy responses can be expressed as

$$\{x(t)\} = \sum_{k=1}^{2p} \{\psi\}_k e^{\lambda_k t} + \sum_{k=2p+1}^{2m} \{N\}_k e^{r_k t} \tag{7}$$

in which the noise is modeled as a combination of $(2m-2p)$ complex exponential functions. Since the value of p, the number of excited modes, is a characteristic of the structural response and not the data analysis process, additional exponential functions are allowed to represent the noise in the math model as m is increased. This results in a higher-order fit for the noise portion of the responses, reducing residuals that would otherwise be included in the signal portion of the responses.

Mode Shape Correlation Constant

When two or more sets of measurements are used in identifying the modal parameters of a test structure, corresponding modes obtained from different identification

Table 1 Model and identification parameters for the four simulated plate tests

Test	No. of measurements	No. of modes	Natural frequency, Hz	Damping ζ, %	rms N/S ratio, %	Allowed NDOF
1	225	2	20.000	1.000	0.0001	300
			30.000	1.000		
2	225	2	30.000	1.000	0.0001	300
			30.000	1.000		
3	225	2	20.000	1.000	200	300
			30.000	1.000		
4	225	30	10.000	1.000	20	300
			11.000	1.000		
			12.000	1.000		
			...	...		
			39.000	1.000		

runs can often be matched by comparing modal frequencies and damping factors. This procedure can lead to mismatching of some modes, however, particularly in regions of high modal density, owing to variance in the identified frequency and damping data. To reduce the possibility of mismatching, mode shape matching can be used in addition to frequency and damping matching.

Let $\{\psi_1\}$ and $\{\psi_2\}$ be two identified (complex) modal vectors from identification runs 1 and 2 that use an overlapping set of measurements. If the modal vectors at the common measurements are denoted by $\{\gamma_1\}$ and $\{\gamma_2\}$, the correlation of the mode shapes can be estimated by the degree of correlation between $\{\gamma_1\}$ and $\{\gamma_2\}$ using a mode shape correlation constant (MSCC) defined as

$$\text{MSCC} = \frac{|\{\gamma_1\}^T\{\gamma_2\}^*|^2}{\{\psi_1\}^T\{\gamma_1\}^*\{\gamma_2\}^T\{\gamma_2\}^*} \tag{8}$$

where T denotes the transpose and * the complex conjugate.

The MSCC between two (complex) modal vectors ranges from 0 for no resemblance to 100% for perfect resemblance. Values between 0 and 100% can be interpreted as the amount of coherent information in the two compared mode shapes. Of course, care should be exercised in using the MSCC information since it can indicate false correlation between overall mode shapes if the number of elements in $\{\gamma_1\}$ and $\{\gamma_2\}$ is small and only portions of the two shapes have some resemblance. Using frequencies and damping factors, together with MSCC, can significantly reduce the possibility of mismatching.

Results and Discussion

In this section, results from several data analyses of responses from two test structures are reported and discussed. The first structure is a NASTRAN-simulated rectangular plate. The other is the long duration exposure facility (LDEF) Space Shuttle payload. The simulated plate results are included to demonstrate typical accuracies which are obtained using simulated free responses from a system with known modal parameters and an overexpanded identification model. The results of the simulation study also help in interpreting and supporting the accuracy of the LDEF experimental results that follow.

Simulated Plate Results

An arbitrary number of NASTRAN mode shapes of a rectangular plate, with arbitrarily assigned natural frequencies, damping factors, and response levels, were used in constructing free-response functions analyzed with the identification algorithm. Four sets of response functions were formed for use in four different identification runs. The first three contained only two modes, with varying assigned modal frequencies and noise-to-signal (N/S) ratios. The number of measurement stations and structural modes in the fourth set were selected to simulate those of an actual large modal survey test. In each set, the response levels of all modes were set equal. Table 1 shows the parameters used in establishing and analyzing each of these simulation data sets. NDOF is the number of computational degrees of freedom allowed in the identification model.

Table 2 Identification results for the four simulated plate tests

Test	Mode No.	Natural frequency, Hz	Damping ζ, %	OAMCF,[a] %	MSCC, %
1	1	20.000	1.000	100	100
	2	30.000	1.000	100	100
2	1	30.000	1.000	100	100
	2	30.001	1.000	100	100
3	1	20.006	2.234	78	98
	2	29.989	1.981	72	98
4	1	9.998	2.100	91	99
	2	11.003	1.812	90	99
	3	11.990	1.746	95	99
	4	13.000	2.047	91	99
	5	14.006	1.492	98	99
	6	14.999	1.879	92	99
	7	16.005	1.505	93	99
	8	17.012	1.255	96	100
	9	17.999	1.615	88	99
	10	19.001	1.691	86	99
	11	20.003	1.435	94	99
	12	20.993	1.365	89	99
	13	22.007	1.260	93	99
	14	22.994	1.385	87	99
	15	23.985	1.334	88	99
	16	24.990	1.498	84	99
	17	26.000	1.194	95	99
	18	27.000	1.447	81	99
	19	28.009	1.314	87	99
	20	29.019	1.195	89	99
	21	30.006	1.373	83	99
	22	31.008	1.113	92	99
	23	32.010	1.394	81	99
	24	33.002	1.447	79	98
	25	34.007	1.299	78	99
	26	35.024	1.332	82	98
	27	36.026	1.387	75	98
	28	37.001	1.458	69	98
	29	37.982	1.459	76	96
	30	39.033	1.301	86	99

[a] All other modes ("noise modes") had an OAMCF of less than 2%.

The identified frequencies and damping factors for all four tests are listed in Table 2. Also included are the overall modal confidence factor[4] (OAMCF) for each identified mode and the MSCC value, showing the correlation between the identified and theoretical mode shapes. The OAMCF value is the percentage of measurement stations whose calculated MCF values for each mode were at least 95% in amplitude and no larger than 10 deg in phase. The OAMCFs for "noise modes" are very small, generally less than 2%.

The results for tests 1 and 2 are included to show that a significantly oversized identification model did not cause singularity or ill-conditioning even in cases of extremely low noise-to-signal ratio (tests 1 and 2) or with almost identical eigenvalues (test 2). Theoretically, for noise-free data, the rank of the $[\Phi]$ matrix is equal to the number of independent modal vectors (twice the number of structural modes) and its determinant is proportional to the differences between the system's eigenvalues.[1] Test 3 demonstrates the effect of using an oversized identification model when processing high noise-to-signal data in reducing errors in the identified parameters. Although the rms noise-to-signal ratio was 200%, negligible error was found in the identified frequencies and mode shapes, and the errors in the damping factor results were bounded. These characteristics also apply to the identification results of the 30-mode simulated data set, test 4.

All of the identified mode shapes are nearly indistinguishable from the theoretical shapes used in forming the simulated response functions. The largest identification error occurred in mode 29, with an MSCC value calculated between the theoretical and identified mode shapes of 96%.

For all four tests, even though the ratio of the number of allowed degrees of freedom in the identification model to the number of structural modes in the responses was very high (150 for tests 1-3 and ten for test 4), no false modes or numerical anomalies resulted in the identifications. All "noise modes" (298 in tests 1-3 and 270 in test 4) had an OAMCF less than 2%.

LDEF Results

The LDEF, shown in Fig. 1, is a 30-ft long, 12-sided cylindrical structure designed to hold 86 experiment trays around its periphery. It will be placed in Earth orbit for extended periods of time to study the effects of space on selected materials and scientific processes mounted in the experiment trays. A large modal survey program was conducted at the Langley Research Center with the structure suspended in a free-free configuration. For these tests the experiment trays were removed from the structure and 142 acceleration measurement stations were used. Data obtained with the structure excited with single-shaker, wide-band random noise in the y (lateral) and z (vertical) directions will be shown. Unit-impulse response functions, obtained by inverse Fourier transformation of acceleration/force frequency response functions, are used in the ITD identification runs. A photograph of the structure with the trays removed, mounted on a transport vehicle, is shown in Fig. 1b. Figure 1c illustrates the positions of the y and z exciters and the location and sensing direction of accelerometers placed on the structure for the vibration tests.

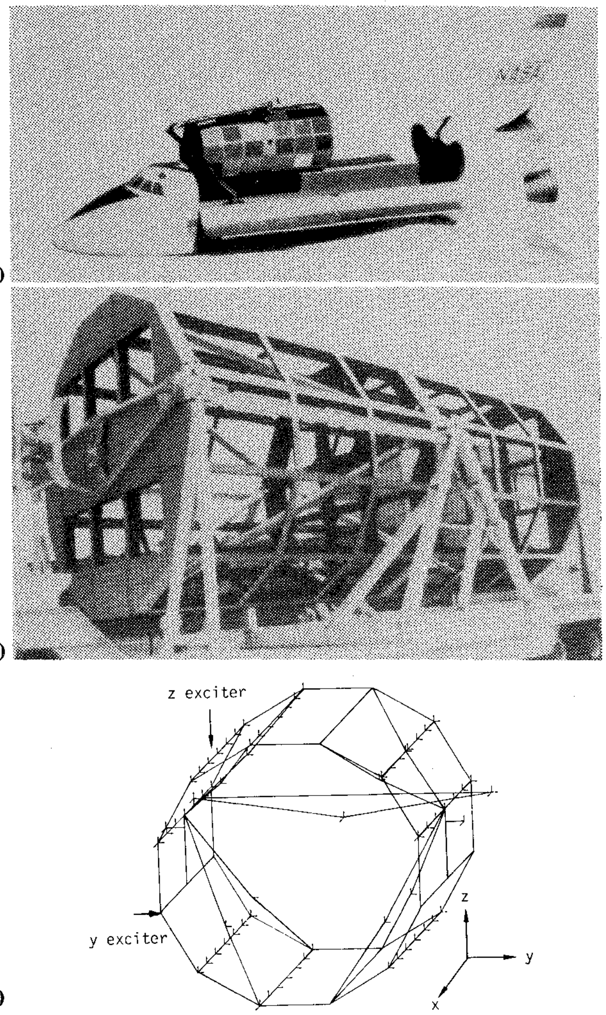
a) b) c)


Fig. 1 Long duration exposure facility (LDEF). a) Typical deployment from the Space Shuttle. b) On transport vehicle with experiment trays removed. c) Measurement positions and two exciter locations used for vibration tests.

Table 3 Test and analysis information for the LDEF identification runs

Run	Force excitation direction	No. of measurement stations	Analysis frequency range, Hz	Allowed NDOF
1	z	142	5-55	1-75 Steps of 1 and 80-200 Steps of 10
2	z	142	5-55	150
3	z	142	5-55	300
4	z	142	19.75-32.25	150
5	y	142	5-55	300
6	z	81 (measurements 1-81)	5-55	81
7	z	81 (measurements 62-142) 142)	5-55	81

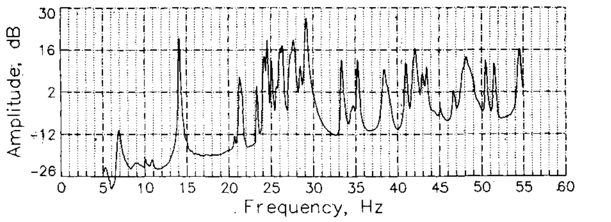


Fig. 2 Average quadrature component of 142 frequency response functions used to form impulse responses for LDEF identification runs 1-3.

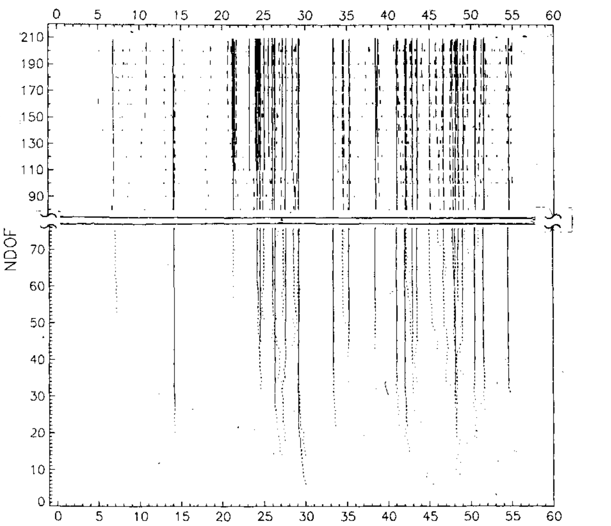


Fig. 3 NDOF-frequency map for LDEF identification run 1.

Results for seven identification runs, using a range of allowed degrees of freedom and two different excitation conditions, will be shown. Table 3 summarizes the test and analysis parameters for each run.

Figure 2 shows the average value of the quadrature components of all 142 frequency response functions for the z-excitation test, over the 5-55-Hz frequency range. The absolute value of each quadrature function was taken prior to averaging, and the result is presented on a logarithmic scale. This "composite" function provides a good indication of both the natural frequencies and relative response levels of the structural modes excited in the z-excitation test. For this plot, the reference decibel level has no special significance.

In run 1, the number of allowed degrees of freedom (NDOF) was incremented from 1 to 75 in steps of 1, and then from 80 to 200 in steps of 10. Figure 3 shows a "map" of the identified modal frequencies as a function of NDOF. The identified frequencies are denoted by vertical line segments at the corresponding frequencies whose heights are proportional to the OAMCF value calculated for each mode. An OAMCF of 100% is represented by a line height equal to the distance between adjacent NDOF values used in the analysis. A continuous vertical line indicates higher confidence in the identified mode, while a dashed line shows lower confidence.

It is of interest to note the order in which modes appear on the map as NDOF is increased. The first mode to appear, near 30 Hz, has the largest average response level, as seen in Fig. 2. The next two modes appear near 27 Hz, followed by ones near 14, 42, and 48 Hz. As NDOF is increased further, some lower level modes near 7 and 21 Hz are identified. As suggested by Eq. (7), the average strength of the "noise modes" decreases when the size of the identification model is expanded, allowing lower strength structural modes to be identified. This behavior is clearly indicated in Fig. 3. As NDOF increases from 80 to 200, more new vertical lines start to form, indicating new identified modes. Based on the results of many other simulated plate experiments,[4] no evidence exists to suggest that these new modes are adverse effects resulting from the expansion of the identification model size used in the identification. The straightness of the vertical lines in this map illustrates the insensitivity of the identified frequencies to higher numbers of degrees of freedom than necessary for initial identification, and the consistency and stability of the analysis process at high NDOF values.

In runs 2 and 3, unit-impulse responses for the 142 stations over the 5-55-Hz frequency range, with single-shaker z excitation of the structure, are used in identifications with 150 and 300 degrees of freedom (DOF), respectively. In run 4, "zoomed" transfer functions over the frequency interval of 19.75 to 32.25 Hz, transformed to the time domain, were analyzed with 150 DOF. For run 5, responses to single-shaker y excitation of the structure, over the 5-55-Hz range, were analyzed with NDOF of 300.

The curves in Figs. 4a-d show the average quadrature component of all 142 frequency response functions obtained for runs 2-5, respectively. The curves consist of 512 equally spaced values each. The diamond symbols placed above the curves denote the frequencies of all ITD-identified modes with an OAMCF of 60% or larger, for each of the four runs. The 60% OAMCF cutoff is arbitrarily selected to single out strongly identified modes. These figures are provided to illustrate three basic results of this study: The strong correlation between ITD-identified modal frequencies and peaks in measured frequency response functions; the identification of modes with low response level as the number of allowed degrees of freedom is increased; and the ability of the identification algorithm to identify modes which are spaced closer in frequency than the resolution of classical Fourier analysis. In these figures the diamond symbols are placed equidistant from the composite quadrature functions at each ITD-identified frequency, and indicate the identified modal frequencies only.

Many interesting comparisons of the results shown in Fig. 4 can be made, some of which are highlighted by circled letters a-d. Near 21.5 Hz, denoted region a, three modes are identified using 150 DOF in Fig. 4a, and four modes are identified using 300 DOF in Fig. 4b. On examining Fig. 4c, in which the resolution of the frequency response function is four times greater than in 4a or 4b, the existence of four distinct response peaks is apparent in region a. In both Figs. 4a and 4b, a mode was identified at 24.0 Hz, denoted by b, where no indication of a structure mode was apparent. Again on examining Fig. 4c, the existence of this mode is just discernible along the ramp of the more strongly excited mode at 24.2 Hz. Region c shows several highly coupled modes identified with the z-excitation response data in Figs. 4a and 4b, but are better separated in the y-excitation data, Fig. 4d. Region d shows two identified modes in Fig. 4d near 43 Hz, where a more-defined response is noted in Fig. 4b.

To study the consistency of the identifications and to demonstrate an application of the MSCC parameter, Figs. 5a and 5b provide "cross-plots" of the ITD-identified modal frequencies and damping factors, respectively, determined from two independent tests of the LDEF: run 3 for z excitation and run 5 for y excitation. Both identifications were run using 300 allowed degrees of freedom. The data shown in these plots represent results of correlating all 300 identified modes ("noise" and structural) from run 3 with all 300 from run 5, using the MSCC parameter. Results for all pairs of modes with a calculated MSCC of 80% or larger are shown. The excellent agreement of identified frequencies shown in Fig. 5a implies not only that consistent mode shapes were determined in two independent tests of the structure, but that the calculation of MSCC values using a large number of measurements (142 for these data) can potentially match identified modes independent of a comparison of identified

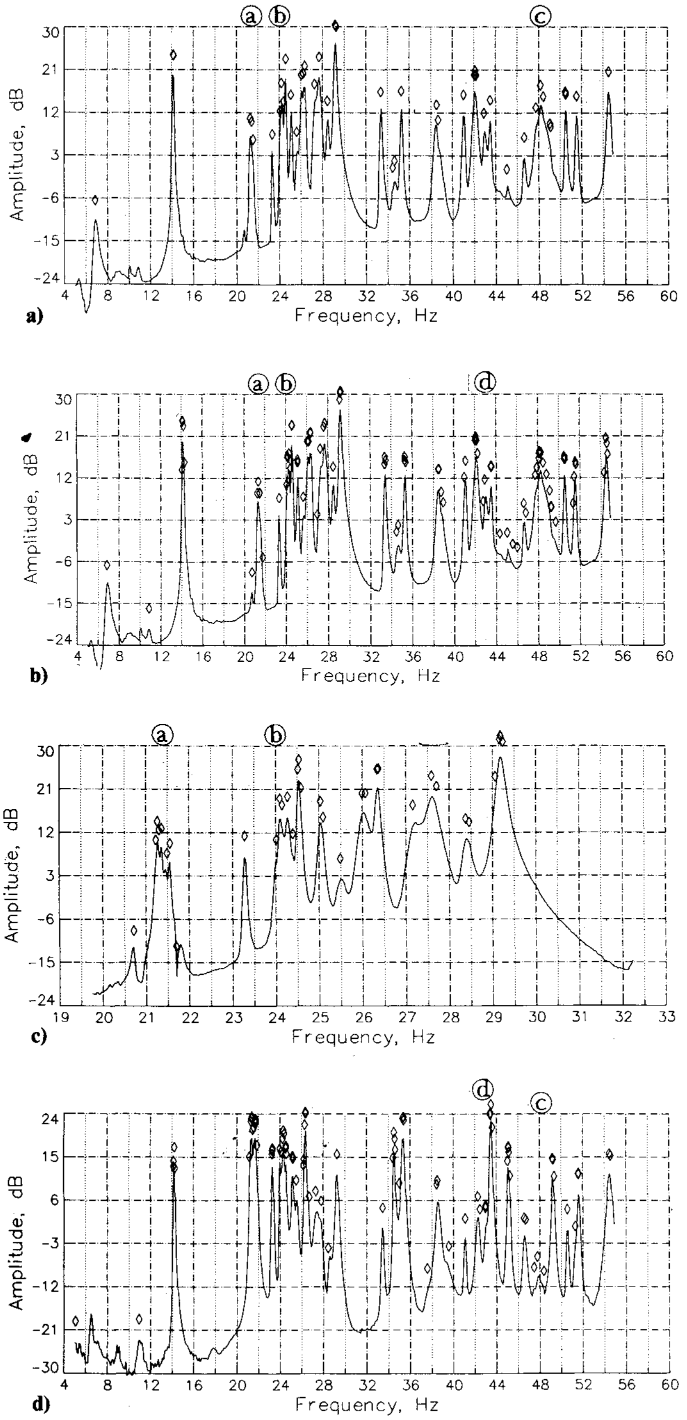


Fig. 4 Comparison of ITD-identified modal frequencies with average quadrature component of frequency response functions used to form impulse responses for LDEF runs 2-5. a) Run 2: NDOF = 150, *z* excitation, 5-55 Hz. b) Run 3: NDOF = 300, *z* excitation, 5-55 Hz. c) Run 4: NDOF = 150, *z* excitation, 19.75-32.25 Hz. d) Run 5: NDOF = 300, *y* excitation, 5-55 Hz.

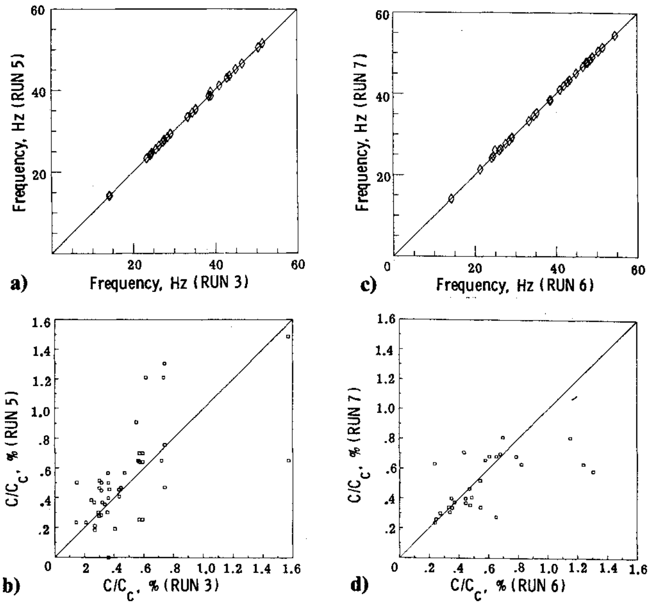


Fig. 5 Cross plots of identified frequencies and damping factors for mode pairs with high MSCC (>80%) using different exciter directions (runs 3 and 5) and overlapping measurements (runs 6 and 7). a) Frequencies: *z* excitation (run 3) vs *y* excitation (run 5). b) Damping factors: for modes shown in a. c) Frequencies: stations 62-81 from run 6 vs stations 62-81 from run 7. d) damping factors: for modes shown in c.

modal frequencies and damping factors. The damping factors identified for each pair of modes whose frequencies are shown in Fig. 5a are included in Fig. 5b.

Identification runs 6 and 7 illustrate a method for matching modes identified in two or more separate runs, using several common measurements in each. This process can be used in analyzing data from a test where limited computer memory restricts the processing for all available response measurements simultaneously. In run 6, stations 1-81 were used with 81 allowed degrees of freedom, and in run 7, stations 62-142, also with 81 DOF. Figures 5c and 5d show the results, in the same format used in Figs. 5a and 5b, of the correlation of two ITD identifications where 20 of the available free-response functions are common in each run. These plots show those mode pairs with MSCC>80%, calculated using only the 20 common elements of the identified mode shapes, and with OAMCF>15%. Several of the modes from run 6 correlated at 80% or higher with more than one mode in run 7 when only the 20 common mode shape elements were used in the calculation of MSCC. In these cases, only the pair of modes whose MSCC was highest is included in Figs. 5c and 5d. In nearly every case, this pairing is the same as that resulting from pairing those modes closest in identified frequency from each of the groups. As in Figs. 5a and 5b, the identified frequencies paired in this manner are almost identical, and the scatter in the corresponding damping factors is small for most modes.

Conclusions

The number of degrees of freedom to be allowed in the ITD identification algorithm can be several times larger than the number of structural modes of vibration excited in the time response functions used for the identification of modal parameters. No adverse effects, in either the accuracy or consistency of identification, resulted from the use of significantly oversized identification models. Such large models are useful in identifying a complete set of modal parameters for all available measurements in one computer analysis. This is valuable for large modal survey tests when a large number of response measurements are obtained. Furthermore, it was found that larger models improved the identification accuracy when noise was present and also allowed the identification of modes of low levels of response. Identifications with identification models of up to 300 DOF proved accurate for data with 200% noise-to-signal ratio and did not result in ill-conditioning for data with infinitesimal noise.

A mode shape correlation constant was used in matching mode shapes obtained from different identification runs, reducing the possibility of mismatching if only frequencies and damping factors are used. This concept was demonstrated in matching experimentally identified modal parameters from two sets of overlapping measurements due to the same excitation, and from different excitation conditions.

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